

# The Decay Properties of the $1^{-+}$ Hybrid State

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Within the framework of the QCD sum rules, we consider the three-point correlation function, work at the limit  $q^2 \rightarrow 0$  and  $m_\pi \rightarrow 0$ , and pick out the singular term  $\sim \frac{1}{q^2}$  to extract the pionic coupling constants of the  $1^{-+}$  hybrid meson. Then we calculate the decay widths of different modes. The decay width of the  $S$ -wave modes  $b_1\pi, f_1\pi$  increases quickly as the hybrid meson mass and decay momentum increase. But for the low mass hybrid meson around 1.6 GeV, the  $P$ -wave decay mode  $\rho\pi$  is very important and its width is around 180 MeV, while the widths of  $\eta\pi$  and  $\eta'\pi$  are strongly suppressed. We suggest the experimental search of  $\pi_1(1600)$  through the decay chains at BESIII:  $e^+e^- \rightarrow J/\psi(\psi') \rightarrow \pi_1 + \gamma$  or  $e^+e^- \rightarrow J/\psi(\psi') \rightarrow \pi_1 + \rho$  where the  $\pi_1$  state can be reconstructed through the decay modes  $\pi_1 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$  or  $\pi_1 \rightarrow f_1(1285)\pi^0$ . It is also interesting to look for  $\pi_1$  using the available BELLE/BABAR data through the process  $e^+e^- \rightarrow \gamma^* \rightarrow \rho\pi_1, b_1\pi_1, \gamma\pi_1$  etc.

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## I. INTRODUCTION

A hybrid meson state contains one valence quark, one valence anti-quark and one valence gluon [1–4], which is in contrast with the  $\bar{q}q$  meson in the conventional quark model. Especially the hybrid states with  $J^{PC} = 0^{--}, 0^{+-}, 1^{--}, 2^{+-}, \dots$  are of particular interests since these exotic quantum numbers arise from the manifest gluon degree of freedom and can not be accessed by the  $\bar{q}q$  meson. Experimental confirmation of the exotic hybrid mesons and glueballs will be a direct test of quantum chromodynamics in the low energy sector. In the past three decades there have been a lot of experimental and theoretical investigations of the hybrid states. However, their nature remains elusive.

Up to now there are three candidates with  $J^{PC} = 1^{-+}$ :  $\pi_1(1400)$ ,  $\pi_1(1600)$  and  $\pi_1(2000)$ . According to PDG, their masses and widths are  $(1376 \pm 17, 300 \pm 40)$  MeV,  $(1653^{+18}_{-15}, 225^{+45}_{-28})$  MeV and  $(2014 \pm 20 \pm 16, 230 \pm 21 \pm 73)$  MeV [5]. Many collaborations reported evidence of the  $1^{-+}$  states such as the CLAS Collaboration, the Crystal Barrel Collaboration, the COMPASS Collaboration, the E862 Collaboration etc.. For example, the  $\pi_1(1400)$  state was observed in the reactions  $\pi^-p \rightarrow \eta\pi^0n$  [6],  $\bar{p}p \rightarrow \pi^0\pi^0\eta$ ,  $\bar{p}n \rightarrow \pi^-\pi^0\eta$  [7],  $\pi^-p \rightarrow \eta\pi^-p$  [8]. The  $\pi_1(1600)$  state was observed in the reactions  $\pi^-p \rightarrow \eta'\pi^-p$  [9],  $\pi^-p \rightarrow \omega\pi^-\pi^0p$  [10] and  $\pi^-p \rightarrow \eta\pi^+\pi^-\pi^-p$  [11]. The  $\pi_1(2015)$  meson was observed in the channels  $\pi^-p \rightarrow \omega\pi^-\pi^0p$  [10] and  $\pi^-p \rightarrow \eta\pi^+\pi^-\pi^-p$  [11]. Some other recent experimental papers include Refs. [12–15]. We note that the Ref. [15] studied the  $\pi^-\pi^-\pi^+$  final state whose data show a significant production of  $\pi_1(1600)$  decaying to  $\rho\pi$ . We also note that the existence of  $\pi_1(2015)$  is still questionable, and it is not included in the most recent PDG 2010 [16].

There have been many theoretical calculations of the hybrid meson mass in literature [17–23]. The  $1^{-+}$  mass extracted from the quenched lattice QCD simulation ranges from 1.74 GeV [28] and 1.8 GeV [29] to 2 GeV [27]. Among the various phenomenological models, the flux tube model is a popular one. Within this framework the glue degree of freedom is modeled by a semi-classical color flux tube. The hybrid meson with the quantum numbers  $J^{PC} = 1^{-+}$  was found to be around 1.9 GeV [25, 26]. The mass from constituent gluon model is around 1.93 GeV [30]. Many authors studied the mass of the hybrid meson using the QCD sum rule formalism [31–38]. In our previous papers, we used the tetraquark currents ( $qq\bar{q}\bar{q}$ ) to study the  $1^{-+}$  mesons using the method of QCD sum rule since both the tetraquark and hybrid interpolating currents may couple to the same states. The extracted mass is around

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1.6 GeV and 2 GeV, quite close to the  $\pi_1(1600)$  and  $\pi_1(2000)$  [39, 40].

To learn more about the hybrid states, it is equally important to study their decay properties besides their mass spectrum. In the flux tube model, a hybrid meson decays as the flux tube breaks. The “ $^3P_0$  pair creation” mechanism is introduced to create a quark-antiquark pair with  $S = 1$ ,  $L = 1$  and total angular momentum  $J = 0$ . With this assumption, the lowest-lying hybrid state with  $J^{PC} = 1^{-+}$  cannot decay into two ground states, such as  $\pi\pi$ ,  $\pi\eta$  and  $\pi\rho$ , etc in the original flux tube model [41]. Later some authors modified this phenomenological model further with the introduction of a new decay vertex in order to study the hybrid meson decay process [42, 43]. With this modification, the contribution of the  $P$ -wave decay mode  $\rho\pi$  is not negligible. There are also other theoretical approaches on the decay, photo- and electro-production of the  $1^{-+}$  hybrid meson [35, 44–52].

In literature, the three-point correlation function was invoked to discuss the decay width of the modes  $\rho\pi$ ,  $\eta\pi$  etc [35, 45], where the sum rules were derived at the symmetric point  $p^2 = (p' + q)^2 = q^2 = -Q^2 > 0$ . Later, the light cone QCD sum rules was employed to calculate the decays of the hybrid mesons by one of the present authors [46, 47].

In order to perform a systematical study of the  $1^{-+}$  hybrid meson we use a different formalism within the framework of QCD sum rule in this paper, which was first developed in Ref. [53]. After calculating the three-point correlation functions using the method of operator product expansion, we ignore the small pion mass term  $m_\pi^2$  in the denominator and pick out the divergent term  $\frac{1}{q^2}$  in the limit  $q^2 \rightarrow 0$  where  $q$  is the pion momentum. After comparing the singular term  $\frac{1}{q^2}$  of the three-point correlation function both at the phenomenological and quark-gluon level, we make Borel transformation of the variables  $p^2, p'^2 = (p + q)^2$ , and extract the coupling constants (such as  $g_{\rho\pi}$ ). Then we calculate the decay widths of both the isovector hybrid states  $\pi_1$  of  $I^G J^{PC} = 1^{-1-+}$  and the isoscalar ones  $\sigma_1$  of  $I^G J^{PC} = 0^{+1-+}$ . The non-vanishing decay modes include  $\pi_1 \rightarrow \rho\pi$ ,  $\eta\pi$ ,  $\eta'\pi$ ,  $b_1\pi$ ,  $f_1\pi$ , and  $\sigma_1 \rightarrow \eta\eta'$ ,  $a_1\pi$ ,  $f_1\eta$ , etc.

Our paper is separated into several sections according to the different decay modes. In Sec. II, we study the decay mode  $\pi_1 \rightarrow \rho\pi$ . In Sec. III, we study the decay modes  $\pi_1 \rightarrow \eta\pi$  and  $\eta'\pi$ . In Sec. IV, we study the decay mode  $\pi_1 \rightarrow b_1\pi$  using the derivative current. In Sec V, we study the decay mode  $\pi_1 \rightarrow f_1\pi$ . In Sec VI, we extend the same formalism to study the isoscalar hybrid state with  $I^G J^{PC} = 0^{+1-+}$ . Sec. VII is the summary. In Appendix A, we study the decay mode  $\pi_1 \rightarrow b_1\pi$ , using the tensor current for  $b_1$ .

## II. THE DECAY MODE $\pi_1 \rightarrow \rho\pi$

### A. Three-Point Correlation Function

For the past decades QCD sum rule has proven to be a very powerful and successful non-perturbative method [56, 57]. In the usual QCD sum rule, we consider the two-point correlation function in order to extract the hadron masses. In this paper we consider the three-point correlation functions in order to study the decay properties of hadrons:

$$T^{A \rightarrow BC}(p, p', q) = \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} \eta^B(x) \eta^C(y) \eta^{A\dagger}(0) | 0 \rangle. \quad (1)$$

The hybrid state  $\pi_1$  has several decay modes, such as  $\rho\pi$ ,  $f_1(1285)\pi$ ,  $b_1(1235)\pi$  etc.. So we need calculate several three-point correlation functions. The procedures are more or less the same. In this section we study the decay mode  $\pi_1 \rightarrow \rho\pi$ . First we show the three currents used in our calculation. The hybrid current with  $J^{PC} = 1^{-+}$  is:

$$\eta_\mu^{\pi_1} = \bar{q}^a \gamma^\nu \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n q^b, \quad (2)$$

where the summation is taken over repeated indices ( $\mu, \nu, \dots$  for Lorentz indices, and  $a, b, \dots$  for color indices). The hybrid state  $\pi_1$  is an isovector state. We study the neutral one, whose quark configurations are  $(\bar{u}u - \bar{d}d)g$ :

$$\eta_\mu \equiv \eta_\mu^{\pi_1^0} = \frac{1}{\sqrt{2}} (\bar{u}^a \gamma_\nu u^b - \bar{d}^a \gamma_\nu d^b) \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n, \quad (3)$$

and it couples to  $\pi_1(1600)$  through

$$\langle 0 | \eta_\mu | \pi_1(p, \lambda) \rangle = \sqrt{2} f_{\pi_1} m_{\pi_1}^3 \epsilon_\mu^\lambda. \quad (4)$$

For the  $\rho$  meson, we use the vector current

$$j_\mu^{\rho^+} = \bar{d} \gamma_\mu u, \quad j_\mu^{\rho^0} = \frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d), \quad j_\mu^{\rho^-} = \bar{u} \gamma_\mu d, \quad (5)$$

and it couples to the vector meson  $\rho$  through

$$\langle 0 | j_\mu^\rho | \rho(p, \lambda) \rangle = m_\rho f_\rho \epsilon_\mu^\lambda. \quad (6)$$

For the pseudoscalar meson  $\pi$ , we use

$$j_5^{\pi^+} = \bar{d}\gamma_5 u, \quad j_5^{\pi^0} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d), \quad j_5^{\pi^-} = \bar{u}\gamma_5 d, \quad (7)$$

and it couples to  $\pi$  through

$$\langle 0 | j_5^\pi | \pi(p) \rangle = f_\pi' = \frac{2i\langle \bar{q}q \rangle}{f_\pi}. \quad (8)$$

Having defined all these interpolating currents and their couplings to the physical hadrons, we can write down the three-point correlation function at the phenomenological side:

$$\begin{aligned} T_{\mu\nu}^{\rho\pi(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} j_\nu^\rho(x) j_5^{\pi^+}(y) \eta_\mu^\dagger(0) | 0 \rangle \\ &= g_{\rho\pi} \epsilon_{\mu'\nu'\alpha\beta} q^\alpha p'^\beta (g_{\mu\mu'} - \frac{p_\mu p_{\mu'}}{m_{\pi_1}^2}) (g_{\nu\nu'} - \frac{p'_\nu p'_{\nu'}}{m_\rho^2}) \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_\rho m_\rho f_\pi'}{(m_{\pi_1}^2 - p^2 - i\epsilon)(m_\rho^2 - p'^2 - i\epsilon)(m_\pi^2 - q^2 - i\epsilon)} \\ &= g_{\rho\pi} \epsilon_{\mu\nu\alpha\beta} q^\alpha p'^\beta \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_\rho m_\rho f_\pi'}{(m_{\pi_1}^2 - p^2 - i\epsilon)(m_\rho^2 - p'^2 - i\epsilon)(m_\pi^2 - q^2 - i\epsilon)}, \end{aligned} \quad (9)$$

where the momenta of  $\pi_1$ ,  $\rho$  and  $\pi$  are  $p_\mu$ ,  $p'_\mu$  and  $q_\mu$ , respectively. This is for the decay of  $\pi_1^0 \rightarrow \rho^- \pi^+$ , and we can obtain the same result for  $\pi_1^0 \rightarrow \rho^+ \pi^-$ . The coupling constant  $g_{\rho\pi}$  is defined as

$$\mathcal{L} = g_{\rho\pi} \epsilon_{\mu\nu\alpha\beta} \pi_1^0 \partial^\mu \pi^+ \partial^\alpha \rho^- \partial^\beta \rho^- + \dots \quad (10)$$

To simplify our calculation, we work at the massless pion pole. Such a formalism was first applied to calculate the pion nucleon coupling constants very successfully decades ago [53–55, 57]. Later the same formalism was employed to discuss various pionic couplings [58]. We first take the small  $\pi$  mass in the denominator to be zero. Then we work at the limit  $q^2 \rightarrow 0$ .  $T_{\mu\nu}^{\rho\pi(\text{PH})}$  is divergent at  $q^2 = 0$  up to the order of  $(q^2)^{-1}$ . Therefore, we will also choose such divergent terms at the QCD side in the next subsection.

## B. Operator Product Expansion

In the previous subsection we have obtained the expression of the three-point correlation function at the phenomenological side. In this subsection we will calculate this at the QCD side using the method of operator product expansion (OPE). To do this, first we insert the three currents into the three-point correlation function. After doing the Wick contractions we obtain:

$$\begin{aligned} &\langle 0 | \mathbb{T} j_\nu^\rho(x) j_5^{\pi^+}(y) \eta_\mu^\dagger(0) | 0 \rangle \\ &= \frac{1}{2\sqrt{2}} \text{Tr} \left( i S_d^{cd}(x-y) \gamma_5 i S_u^{da}(y) \gamma_{\mu'} i S_u^{bc}(-x) \gamma_\nu \right) \lambda_{ab}^n g_s G_{\mu\mu'}^n(0) \\ &\quad - \frac{1}{2\sqrt{2}} \text{Tr} \left( i S_d^{ca}(x) \gamma_{\mu'} i S_d^{bd}(-y) \gamma_5 i S_u^{dc}(y-x) \gamma_\nu \right) \lambda_{ab}^n g_s G_{\mu\mu'}^n(0). \end{aligned} \quad (11)$$

Inside this expression,  $S_u$  and  $S_d$  are the quark propagators for *up* and *down* quark, respectively. In the presence of quark and gluon condensates their expressions are:

$$\begin{aligned} i S^{ab}(x) &\equiv \langle 0 | T[q^a(x) q^b(0)] | 0 \rangle \\ &= \frac{i\delta^{ab}}{2\pi^2 x^4} \hat{x} + \frac{i}{32\pi^2} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \hat{x} + \hat{x} \sigma^{\mu\nu}) - \frac{\delta^{ab}}{12} \langle \bar{q}q \rangle + \frac{\delta^{ab} x^2}{192} \langle g_c \bar{q} \sigma G q \rangle, \end{aligned} \quad (12)$$

where the masses of *up* and *down* quarks have been neglected already.

The OPE calculation is largely simplified when we work at the pion pole and only choose the terms divergent at the  $q^2 \rightarrow 0$  limit:

$$T_{\mu\nu}^{\rho\pi(\text{OPE})}(p, p', q) = \frac{\epsilon_{\mu\nu\alpha\beta} q^\alpha p'^\beta}{q^2} \left( \frac{\langle g_s \bar{q} \sigma G q \rangle}{6\sqrt{2}} \left( \frac{3}{p^2} + \frac{1}{p'^2} \right) - \frac{\langle \bar{q}q \rangle \langle g_s^2 G G \rangle}{18\sqrt{2}} \left( \frac{1}{p^4} + \frac{1}{p'^4} \right) \right). \quad (13)$$

We note here that the tri-gluon condensate  $\langle g_s^3 f G G G \rangle$  as well as the  $\alpha_s$  correction vanishes in this case.

### C. Numerical Analysis

In our numerical analysis, we use the following values for various condensates and  $m_s$  at 1 GeV and  $\alpha_s$  at 1.7 GeV [59–64]:

$$\begin{aligned}
\langle \bar{q}q \rangle &= -(0.240 \text{ GeV})^3, \\
\langle \bar{s}s \rangle &= -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3, \\
\langle g_s^2 GG \rangle &= (0.48 \pm 0.14) \text{ GeV}^4, \\
\langle g_s^3 GGG \rangle &= 0.045 \text{ GeV}^6, \\
\langle g_s \bar{q} \sigma Gq \rangle &= -M_0^2 \times \langle \bar{q}q \rangle, \\
M_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2, \\
m_s(1 \text{ GeV}) &= 125 \pm 20 \text{ MeV}, \\
\alpha_s(1.7 \text{ GeV}) &= 0.328 \pm 0.03 \pm 0.025.
\end{aligned} \tag{14}$$

There is a minus sign in the definition of the mixed condensate  $\langle g_s \bar{q} \sigma Gq \rangle$ , which is different from that used in some other QCD sum rule studies. This difference just comes from the definition of coupling constant  $g_s$  [59]. We use the following values for the initial and final states [47, 57, 65]:

$$\begin{aligned}
m_{\pi_1} &= 1.6 \text{ GeV}, f_{\pi_1} = 0.026 \text{ GeV}, \\
m_\pi &= 140 \text{ MeV}, f'_\pi = \frac{2i\langle \bar{q}q \rangle}{f_\pi} = \frac{2i(-243 \text{ MeV})^3}{131 \text{ MeV}}, \\
m_\rho &= 770 \text{ MeV}, f_\rho = 220 \text{ MeV}.
\end{aligned} \tag{15}$$

We can compare the three-point correlation function at the phenomenological side Eq. (9) and at the QCD side Eq. (13) in order to calculate the coupling constant  $g_{\rho\pi}$ . Due to  $p = p' + q$ , we have several different choices for the QCD sum rules analysis. Here we use  $p'_\mu$  and  $q_\mu$ ,  $p^2$ ,  $p'^2$  and  $q^2$  instead of  $p_\mu$ ,  $p \cdot p'$ ,  $p \cdot q$  and  $p' \cdot q$ .

In the QCD sum rule, we use the Borel transformation to suppress the high order contributions. We use  $T$  to denote the Borel Mass  $M_B$ . Here we have two types of Borel transformation:

1. Assume  $p^2 = p'^2$  and perform the Borel transformation once,  $\mathcal{B}(p^2 = p'^2 \rightarrow T^2)$ ;
2. Perform the Borel transformation twice,  $\mathcal{B}(p^2 \rightarrow T_1^2, p'^2 \rightarrow T_2^2)$ , and then assume  $T_1 = T_2 = T$ .

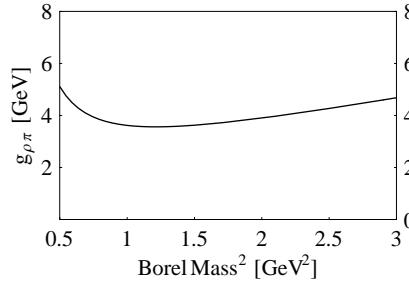


FIG. 1: The coupling constant  $g_{\rho\pi}$  as a function of  $M_B^2$ , calculated using Eq. (16).

For the decay mode  $\rho\pi$ , we employ the first type of Borel transformation. We compare Eqs. (9) and (13), assume  $p^2 = p'^2$  and perform the Borel transformation once,  $\mathcal{B}(p^2 = p'^2 \rightarrow T^2)$  to arrive at

$$-g_{\rho\pi} \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_\rho m_\rho f'_\pi}{m_\rho^2 - m_{\pi_1}^2} \left( e^{-m_{\pi_1}^2/T^2} - e^{-m_\rho^2/T^2} \right) = -\frac{2\langle \bar{q}q \rangle \langle g_s^2 GG \rangle}{3\sqrt{2}} - \frac{\langle \bar{q}q \rangle \langle g_s^3 GGG \rangle}{9\sqrt{2}} \frac{1}{T^2}. \tag{16}$$

Using this equation, the coupling constant  $g_{\rho\pi}$  can be calculated as a function of  $M_B$ , as shown in Fig. 1. The result is around 4 GeV<sup>-1</sup>. The formula of the decay width reads

$$\Gamma(\pi_1^0 \rightarrow \rho^+ \pi^- + \rho^- \pi^+) = 2 \times \frac{g_{\rho\pi}^2}{12\pi} |\vec{q}_\pi|^3, \tag{17}$$

where  $\vec{q}_\pi$  is the momentum of the final state  $\pi$ :

$$|\vec{q}_\pi| = \frac{[(m_{\pi_1}^2 - (m_\rho + m_\pi)^2)(m_{\pi_1}^2 - (m_\rho - m_\pi)^2)]^{1/2}}{2m_{\pi_1}}. \quad (18)$$

The decay width of  $\pi_1 \rightarrow \rho\pi$  is around 180 MeV.

### III. THE DECAY MODES $\pi_1 \rightarrow \eta\pi, \eta'\pi$

#### A. Three-Point Correlation Function

In this section we study the decay modes  $\pi_1 \rightarrow \eta\pi$  and  $\pi_1 \rightarrow \eta'\pi$ . The interpolating current for the  $\eta$  meson is

$$j^\eta = \cos \theta_P j^{\eta_8} - \sin \theta_P j^{\eta_0}, \quad (19)$$

where the mixing angle is  $\theta_P = -17^\circ$  [69], and it couples to  $\eta$  through

$$\langle 0 | j^\eta | \eta(p) \rangle = \lambda_\eta. \quad (20)$$

The  $SU(3)$  octet and singlet currents in Eq. (19) are

$$\begin{aligned} j^{\eta_8} &= \frac{1}{\sqrt{6}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s), \\ j^{\eta_0} &= \frac{1}{\sqrt{3}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s). \end{aligned} \quad (21)$$

The interpolating current for the  $\eta'$  meson is

$$j^{\eta'} = \sin \theta_P j^{\eta_8} + \cos \theta_P j^{\eta_0}, \quad (22)$$

and it couples to  $\eta'$  through

$$\langle 0 | j^{\eta'} | \eta'(p) \rangle = \lambda_{\eta'}. \quad (23)$$

The three-point correlation function for the decay mode  $\pi_1 \rightarrow \eta\pi$  is:

$$\begin{aligned} T_\mu^{\eta\pi(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} j^\eta(x) j^\pi(y) \eta_\mu^\dagger(0) | 0 \rangle \\ &= -(g_{\eta\pi}^A q_\alpha + g_{\eta\pi}^B p'_\alpha)(g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_{\pi_1}^2}) \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 \lambda_\eta f'_\pi}{(m_{\pi_1}^2 - p^2)(m_\eta^2 - p'^2)(m_\pi^2 - q^2)}, \\ &= -(g_{\eta\pi}^A q_\mu + g_{\eta\pi}^B p'_\mu - g_{\eta\pi}^A \frac{p_\mu p \cdot q}{m_{\pi_1}^2} - g_{\eta\pi}^B \frac{p_\mu p \cdot p'}{m_{\pi_1}^2}) \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 \lambda_\eta f'_\pi}{(m_{\pi_1}^2 - p^2)(m_\eta^2 - p'^2)(m_\pi^2 - q^2)}, \end{aligned} \quad (24)$$

where the coupling constants  $g_{\eta\pi}^A$  and  $g_{\eta\pi}^B$  are defined in the following Lagrangian

$$\mathcal{L} = i g_{\eta\pi}^A \pi_1^\mu (\partial_\mu \pi) \eta + i g_{\eta\pi}^B \pi_1^\mu (\partial_\mu \eta) \pi. \quad (25)$$

In order to calculate the three-point correlation function  $T_\mu^{\eta\pi(\text{PH})}$  at the QCD side, once again we work at the pion pole and only choose the terms divergent at the  $q^2 \rightarrow 0$  limit at the quark and gluon level. When  $q^2 \rightarrow 0$ , we obtain

$$\begin{aligned} q^2 T_\mu^{\eta\pi(\text{OPE})}(p, p', q) &\rightarrow X(\theta) \times \left( q_\mu(p^2 - p'^2) \frac{\langle g_s^2 GG \rangle}{24\sqrt{2}\pi^2} \int_0^\infty e^{\tau_1 p^2 + \tau_2 p'^2} \frac{\tau_2}{2(\tau_1 + \tau_2)^2} d\tau_1 d\tau_2 \right. \\ &\quad + \frac{\langle g_s^3 f GGG \rangle}{96\sqrt{2}\pi^2} \int_0^\infty e^{\tau_1 p^2 + \tau_2 p'^2} \times \left( q_\mu \frac{2\tau_1}{(\tau_1 + \tau_2)^2} + p'_\mu \frac{-\tau_2}{(\tau_1 + \tau_2)^2} \right. \\ &\quad \left. + q_\mu(p^2 - p'^2) \frac{-(\tau_1^2 + \tau_2^2)\tau_2}{(\tau_1 + \tau_2)^3} + p'_\mu(p^2 - p'^2) \frac{-\tau_2^2}{(\tau_1 + \tau_2)^2} \right) d\tau_1 d\tau_2 \\ &\quad \left. - \frac{\langle g_s^3 f GGG \rangle}{48\sqrt{2}\pi^2} \frac{p^\mu}{p^2} \right), \end{aligned} \quad (26)$$

where  $X(\theta) = \frac{1}{\sqrt{3}} \cos \theta_P - \sqrt{\frac{2}{3}} \sin \theta_P$ . In the calculations, we have used the following equation [67]

$$\int \frac{e^{ip'x} e^{iqy} d^4x d^4y}{(x-y)^{2l} y^{2m} x^{2n}} = \frac{(-1)^{l+m+n+1} \pi^4}{4^{l+m+n-4} l! m! n!} \int_0^\infty \frac{e^{\tau_1 p^2 + \tau_2 p'^2 + \tau_3 q^2} d\tau_1^l d\tau_2^m d\tau_3^n}{(\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_3 \tau_1)^{l+m+n-2}}. \quad (27)$$

We find that only when  $i \geq j$ , the term

$$\int_0^\infty \frac{e^{\tau_3 q^2} \tau_3^i}{(\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_3 \tau_1)^j} d\tau_3,$$

is divergent at  $q^2 \rightarrow 0$  (it is up to the order of  $(q^2)^{j-i-1}$ ).

## B. Numerical Analysis

To perform the numerical analysis, we use the following values [65]

$$\begin{aligned} m_\eta &= 0.547 \text{GeV}, \lambda_\eta = 0.23 \text{GeV}^2, \\ m_{\eta'} &= 0.958 \text{GeV}, \lambda_{\eta'} = 0.33 \text{GeV}^2. \end{aligned} \quad (28)$$

There are two independent Lorentz structures,  $q_\mu$  and  $p'_\mu$ . We will use both of them to perform the QCD sum rule analysis.

### 1. Lorentz structure $q_\mu$

First we choose the Lorentz structure  $q_\mu$ . If we perform the Borel transformation once, the condensate  $\langle g_s^2 GG \rangle$  vanishes, and only the condensate  $\langle g_s^3 fGGG \rangle$  survives. Consequently, the coupling constants  $g_{\eta\pi}$  is tiny.

If we perform the Borel transformation twice, the condensate  $\langle g_s^2 GG \rangle$  does not vanish, and we obtain

$$\begin{aligned} & (g_{\eta\pi}^A - g_{\eta\pi}^B) \sqrt{2} f_{\pi_1} m_{\pi_1}^3 \lambda_\eta f'_\pi \left( \frac{1}{2} e^{-m_{\pi_1}^2/T_1^2} e^{-m_\eta^2/T_2^2} + \frac{m_\eta^2}{2m_{\pi_1}^2} e^{-m_{\pi_1}^2/T_1^2} e^{-m_\eta^2/T_2^2} \right) \\ &= \frac{X(\theta) \langle g_s^2 GG \rangle}{24\sqrt{2}\pi^2} \left( -\left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{\tau_2}{2(\tau_1 + \tau_2)^2} \right) \\ &+ \frac{X(\theta) \langle g_s^3 fGGG \rangle}{96\sqrt{2}\pi^2} \left( \frac{2\tau_1}{(\tau_1 + \tau_2)^2} + \left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{(\tau_1^2 + \tau_2^2)\tau_2}{(\tau_1 + \tau_2)^3} \right) \Big|_{\tau_1=1/T_1^2, \tau_2=1/T_2^2}. \end{aligned} \quad (29)$$

Using Eq. (29) we can calculate  $(g_{\eta\pi}^A - g_{\eta\pi}^B)$ . The coupling constant  $(g_{\eta'\pi}^A - g_{\eta'\pi}^B)$  can be obtained similarly. They

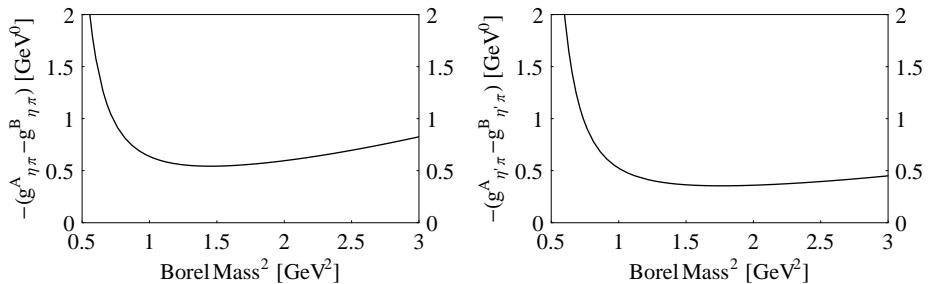


FIG. 2: The coupling constant  $g_{\eta\pi}$  and  $g_{\eta'\pi}$  as functions of  $M_B^2$ . The left figure is for  $(g_{\eta\pi}^A - g_{\eta\pi}^B)$ , calculated using Eq. (29), and the right figure is for  $(g_{\eta'\pi}^A - g_{\eta'\pi}^B)$ . The Lorentz structure here is  $q_\mu$ .

are functions of  $M_B$ , as shown in Fig. 2. The L.H.S. is for  $(g_{\eta\pi}^A - g_{\eta\pi}^B)$ , and the result is around -0.7; the R.H.S is for  $(g_{\eta'\pi}^A - g_{\eta'\pi}^B)$ , and the result is around -0.4. Using the formula of the decay width

$$\Gamma(\pi_1^0 \rightarrow \eta\pi^0) = \frac{|g_{\eta\pi}^A - g_{\eta\pi}^B|^2 |\vec{q}_\pi|^3}{24\pi m_{\pi_1}^2}, \quad (30)$$

the decay width of  $\pi_1 \rightarrow \eta\pi$  is around 0.9 MeV, and the one of  $\pi_1 \rightarrow \eta'\pi$  is about 0.1 MeV.

## 2. Lorentz structure $p'_\mu$

Next we move on to the Lorentz structure  $p'_\mu$ . After performing the Borel transformation twice, we obtain

$$\begin{aligned} & -(g_{\eta\pi}^A - g_{\eta\pi}^B)\sqrt{2}f_{\pi_1}m_{\pi_1}^3\lambda_\eta f'_\pi\left(\frac{1}{2}e^{-m_{\pi_1}^2/T_1^2}e^{-m_\eta^2/T_2^2} - \frac{m_\eta^2}{2m_{\pi_1}^2}e^{-m_{\pi_1}^2/T_1^2}e^{-m_\eta^2/T_2^2}\right) \\ &= \frac{X(\theta)\langle g_s^3 f G G G \rangle}{96\sqrt{2}\pi^2}\left(\frac{-\tau_2}{(\tau_1 + \tau_2)^2} + \left(\frac{\partial}{\partial\tau_1} - \frac{\partial}{\partial\tau_2}\right)\frac{\tau_2^2}{(\tau_1 + \tau_2)^2}\right)\Big|_{\tau_1=1/T_1^2, \tau_2=1/T_2^2}. \end{aligned} \quad (31)$$

However, the condensate  $\langle g_s^2 G G \rangle$  vanishes here, and the obtained result is very small (around -0.05 for  $(g_{\eta\pi}^A - g_{\eta\pi}^B)$ ).

## IV. THE DECAY MODE $\pi_1 \rightarrow b_1(1235)\pi$

### A. Three-Point Correlation Function

In this section we study the decay mode  $\pi_1 \rightarrow b_1(1235)\pi$ . For the  $b_1(1235)$  meson, we use the following current

$$j_\mu^{b_1^+} = (\bar{d}\overleftrightarrow{D}_\mu\gamma_5 u), j_\mu^{b_1^0} = \frac{1}{\sqrt{2}}(\bar{u}\overleftrightarrow{D}_\mu\gamma_5 u - \bar{d}\overleftrightarrow{D}_\mu\gamma_5 d), j_\mu^{b_1^-} = (\bar{u}\overleftrightarrow{D}_\mu\gamma_5 d), \quad (32)$$

and it couples to  $b_1(1235)$  through [57]

$$\langle 0|j_\mu^{b_1}|b_1(1235)(p, \lambda)\rangle = f_{b_1}\epsilon_\mu^\lambda. \quad (33)$$

Then we can write down the three-point correlation function at the phenomenological side:

$$\begin{aligned} T_{\mu\nu}^{b_1\pi(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0|\mathbb{T}j_\nu^{b_1^-}(x)j_\nu^{\pi^+}(y)\eta_\mu^\dagger(0)|0\rangle \\ &= (g_{\mu\mu'} - \frac{p_\mu p_{\mu'}}{m_{\pi_1}^2})(g_{\nu\nu'} - \frac{p'_\nu p'_{\nu'}}{m_{b_1}^2}) \frac{\sqrt{2}f_{\pi_1}m_{\pi_1}^3f_{b_1}f'_\pi}{(m_{\pi_1}^2 - p^2)(m_{b_1}^2 - p'^2)(m_\pi^2 - q^2)} \\ &\quad \times \left( g_{b_1\pi}^S g_{\mu'\nu'} + g_1^D((p \cdot p')g_{\mu'\nu'} - p'_{\mu'}p_{\nu'}) + g_2^D((p \cdot q)g_{\mu'\nu'} - q_{\mu'}p_{\nu'}) \right. \\ &\quad \left. + g_3^D((p' \cdot q)g_{\mu'\nu'} - p'_{\mu'}q_{\nu'}) + g_4^D(q^2g_{\mu'\nu'} - q_{\mu'}q_{\nu'}) \right). \end{aligned} \quad (34)$$

This is for the decay mode  $\pi_1^0 \rightarrow b_1^- \pi^+$ . The same result holds for  $\pi_1^0 \rightarrow b_1^+ \pi^-$ . The coupling constants  $g_{b_1\pi}^S$  and  $g_i^D$  ( $i=1,2,3,4$ ) are defined through the following effective Lagrangians:

$$\mathcal{L}_S = g_{b_1\pi}^S \vec{\pi}_{1\alpha} \times \vec{b}_{1\beta} \cdot \vec{\pi} g^{\alpha\beta}, \quad (35)$$

$$\begin{aligned} \mathcal{L}_D &= +g_1^D(\partial_\alpha \vec{\pi}_{1\beta} - \partial_\beta \vec{\pi}_{1\alpha}) \times \partial^\alpha \vec{b}_{1\beta}^\beta \cdot \vec{\pi} \\ &\quad + g_2^D(\partial_\alpha \vec{\pi}_{1\beta} - \partial_\beta \vec{\pi}_{1\alpha}) \times \vec{b}_{1\beta}^\beta \cdot \partial^\alpha \vec{\pi} \\ &\quad + g_3^D \vec{\pi}_{1\beta}^\beta \times (\partial_\alpha \vec{b}_{1\beta} - \partial_\beta \vec{b}_{1\alpha}) \cdot \partial^\alpha \vec{\pi} \\ &\quad + g_4^D(\vec{\pi}_{1\beta} \times \vec{b}_{1\beta}^\beta \cdot \partial_\alpha \partial^\alpha \vec{\pi} - \vec{\pi}_{1\beta} \times \vec{b}_{1\alpha} \cdot \partial^\alpha \partial^\beta \vec{\pi}), \end{aligned} \quad (36)$$

Once again, the OPE calculation is largely simplified when we work at the pion pole and only choose those divergent

terms at  $q^2 \rightarrow 0$ . When  $q^2 \rightarrow 0$ , we obtain

$$\begin{aligned}
q^2 T_{\mu\nu}^{b_1\pi(\text{OPE})}(p, p', q) \rightarrow & \frac{\langle g_s^2 GG \rangle}{48\sqrt{2}\pi^2} \int_0^\infty e^{\tau_1 p^2 + \tau_2 p'^2} \times \left( -q_\mu q_\nu \frac{\tau_1}{(\tau_1 + \tau_2)^3} \right. \\
& + q_\mu q_\nu (p^2 - p'^2) \frac{\tau_1 \tau_2}{(\tau_1 + \tau_2)^3} + q_\mu p'_\nu (p^2 - p'^2) \frac{\tau_2^2}{(\tau_1 + \tau_2)^3} \Big) d\tau_1 d\tau_2 \\
& + \frac{\langle g_s^3 fGGG \rangle}{192\sqrt{2}\pi^2} \int_0^\infty e^{\tau_1 p^2 + \tau_2 p'^2} \times \left( g_{\mu\nu} \frac{\tau_1}{(\tau_1 + \tau_2)^3} + q_\mu q_\nu \frac{6\tau_1^3 + 6\tau_1^2\tau_2 + 6\tau_1\tau_2^2}{(\tau_1 + \tau_2)^4} \right. \\
& + q_\mu p'_\nu \frac{6\tau_1\tau_2 + 4\tau_2^2}{(\tau_1 + \tau_2)^3} + p'_\mu q_\nu \frac{-4\tau_2}{(\tau_1 + \tau_2)^2} + p'_\mu p'_\nu \frac{-2\tau_2^2}{(\tau_1 + \tau_2)^3} \\
& + g_{\mu\nu} (p^2 - p'^2) \frac{\tau_2}{(\tau_1 + \tau_2)^2} + q_\mu q_\nu (p^2 - p'^2) \frac{(-2\tau_1^2 + 2\tau_1\tau_2 - 2\tau_2^2)\tau_1\tau_2}{(\tau_1 + \tau_2)^4} \\
& + q_\mu p'_\nu (p^2 - p'^2) \frac{(4\tau_1 - 2\tau_2)\tau_2^3}{(\tau_1 + \tau_2)^4} + p'_\mu q_\nu (p^2 - p'^2) \frac{-2\tau_1\tau_2^2}{(\tau_1 + \tau_2)^3} \\
& \left. + p'_\mu p'_\nu (p^2 - p'^2) \frac{-2\tau_2^3}{(\tau_1 + \tau_2)^3} \right) d\tau_1 d\tau_2 - \frac{\langle g_s^3 fGGG \rangle}{48\sqrt{2}\pi^2} \frac{p_\mu p_\nu}{p^2}.
\end{aligned} \tag{37}$$

We note that we do not include the higher order  $\alpha_s$  corrections such as  $\alpha_s \langle g_s^2 GG \rangle$ , etc.

## B. Numerical Analysis

For numerical analysis we use the following values:

$$m_{b_1} = 1.235 \text{ GeV}, f_{b_1}(2 \text{ GeV}) = 0.18 \text{ GeV}^3. \tag{38}$$

where  $f_{b_1}$  is obtained by using Eq. (A.20) in Ref. [57] when assuming  $m_{b_1} = 1235$  MeV.

In order to solve all the five coupling constants  $g_{b_1\pi}^S$  and  $g_i^D$  ( $i=1,2,3,4$ ), we need to find five equations. Comparing Eqs. (34) and (37), we find there are just five Lorentz structures which can be used to obtain five equations and solve these coupling constants. They are  $g_{\mu\nu}$ ,  $p'_\mu p_\nu$ ,  $q_\mu p_\nu$ ,  $p'_\mu q_\nu$ , and  $q_\mu q_\nu$ . Among them,  $p'_\mu q_\nu$  and  $q_\mu q_\nu$  lead to the OPEs which are much larger than those obtained using others. Consequently,  $g_2^D$  and  $g_4^D$  are also much larger than other coupling constants. Using this fact we can simply our calculations a lot.

### 1. Lorentz Structure $g_{\mu\nu}$

First we choose the Lorentz Structure  $g_{\mu\nu}$ . After performing the Borel transformation once, we obtain

$$\begin{aligned}
-g_{b_1\pi}^S \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_b f'_\pi}{m_{b_1}^2 - m_{\pi_1}^2} \left( e^{-m_{\pi_1}^2/T^2} - e^{-m_{b_1}^2/T^2} \right) - g_1^D \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_b f'_\pi}{m_{b_1}^2 - m_{\pi_1}^2} \left( m_{\pi_1}^2 e^{-m_{\pi_1}^2/T^2} - m_{b_1}^2 e^{-m_{b_1}^2/T^2} \right) \\
= -\frac{\langle g_s^3 fGGG \rangle}{384\sqrt{2}\pi^2} T.
\end{aligned} \tag{39}$$

We find that the OPE part only contains the condensate  $\langle g_s^3 fGGG \rangle$ , and so the numerical result turns out to be quite small. Therefore, we will only keep  $g_{\mu\nu}$  term and omit other terms coming from  $g_{\mu'\nu'}(g_{\mu\mu'} - \frac{p_\mu p_{\mu'}}{m_{\pi_1}^2})(g_{\nu'\nu} - \frac{p'_\nu p'_{\nu'}}{m_{b_1}^2})$ , etc. to simply our calculation. The three-point correlation function (34) becomes

$$\begin{aligned}
T_{\mu\nu}^{b_1\pi(\text{PH})}(p, p', q) = & \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_b f'_\pi}{(m_{\pi_1}^2 - p^2)(m_{b_1}^2 - p'^2)(m_\pi^2 - q^2)} \times \left( \right. \\
& g_{\mu\nu} (g_{b_1\pi}^S + g_1^D(p \cdot p') + g_2^D(p \cdot q) + g_3^D(p' \cdot q) + g_4^D q^2) \\
& \left. - g_1^D p'_\mu p'_\nu - g_2^D q_\mu p'_\nu - (g_1^D + g_3^D) p'_\mu q_\nu - (g_2^D + g_4^D) q_\mu q_\nu + \dots \right).
\end{aligned} \tag{40}$$

Using this three-point correlation function, we will show in the next subsection that  $g_1^D$  is zero because the OPE with the Lorentz structure  $p'_\mu p'_\nu$  vanishes. Then after some calculations, we find that the coupling constant  $g_{b_1\pi}^S$  is around 0.02 GeV, which is much smaller compared with the coupling constants  $g_2^D$  and  $g_4^D$ .

## 2. Lorentz Structures $p'_\mu p'_\nu$ and $p'_\mu q_\nu$

For the Lorentz structures  $p'_\mu p'_\nu$  and  $p'_\mu q_\nu$ , the OPE side vanishes, and so we simply obtain

$$g_1^D = 0, \text{ and } g_3^D = 0. \quad (41)$$

## 3. Lorentz Structure $q_\mu p'_\nu$

In this subsection we choose the Lorentz structure  $q_\mu p'_\nu$ . After performing the Borel transformation twice, we obtain

$$\begin{aligned} g_2^D \sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{b_1} f'_\pi e^{-m_{\pi_1}^2/T_1^2} e^{-m_{b_1}^2/T_2^2} &= \frac{\langle g_s^2 GG \rangle}{48\sqrt{2}\pi^2} \left( -\left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{\tau_2^2}{(\tau_1 + \tau_2)^3} \right) \\ &+ \frac{\langle g_s^3 fGGG \rangle}{192\sqrt{2}\pi^2} \left( \frac{6\tau_1\tau_2 + 4\tau_2^2}{(\tau_1 + \tau_2)^3} - \left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{(4\tau_1 - 2\tau_2)\tau_2^3}{(\tau_1 + \tau_2)^4} \right) \Big|_{\tau_1=1/T_1^2, \tau_2=1/T_2^2}. \end{aligned} \quad (42)$$

In Fig. 3 we show the coupling constant  $g_2^D$  obtained using this equation, which is around  $-1.2 \text{ GeV}^{-1}$ .

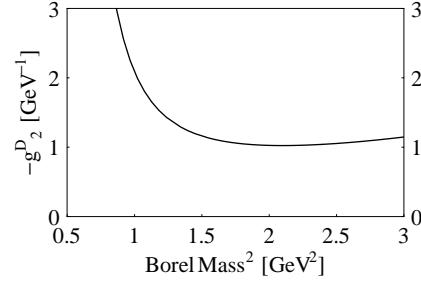


FIG. 3: The coupling constant  $g_2^D$  as a function of  $M_B^2$ , obtained using Eq. (42). The Lorentz structure is  $q_\mu p'_\nu$ .

## 4. Lorentz Structure $q_\mu q_\nu$

In this subsection we choose the Lorentz structure  $q_\mu q_\nu$ . After performing the Borel transformation once, we obtain

$$(g_2^D + g_4^D) \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{b_1} f'_\pi}{m_{b_1}^2 - m_{\pi_1}^2} \left( e^{-m_{\pi_1}^2/T^2} - e^{-m_{b_1}^2/T^2} \right) = \frac{\langle g_s^2 GG \rangle}{96\sqrt{2}\pi^2} T^2 + \frac{\langle g_s^3 fGGG \rangle}{128\sqrt{2}\pi^2}. \quad (43)$$

We can also perform the Borel transformation twice, and then obtain

$$\begin{aligned} (g_2^D + g_4^D) \sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{b_1} f'_\pi e^{-m_{\pi_1}^2/T_1^2} e^{-m_{b_1}^2/T_2^2} &= \frac{\langle g_s^2 GG \rangle}{48\sqrt{2}\pi^2} \left( -\frac{\tau_1}{(\tau_1 + \tau_2)^3} - \left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{\tau_1\tau_2}{(\tau_1 + \tau_2)^3} \right) + \frac{\langle g_s^3 fGGG \rangle}{192\sqrt{2}\pi^2} \left( \frac{6\tau_1^3 + 6\tau_1^2\tau_2 + 6\tau_1\tau_2^2}{(\tau_1 + \tau_2)^4} \right. \\ &\quad \left. - \left( \frac{\partial}{\partial \tau_1} - \frac{\partial}{\partial \tau_2} \right) \frac{(-2\tau_1^2 + 2\tau_1\tau_2 - 2\tau_2^2)\tau_1\tau_2}{(\tau_1 + \tau_2)^4} \right) \Big|_{\tau_1=1/T_1^2, \tau_2=1/T_2^2}. \end{aligned} \quad (44)$$

The quantity  $g_2^D + g_4^D$  is a function of the Borel mass, as shown in Fig. 4. The left and right figures are obtained using Eqs. (43) and (44), respectively. We obtain that  $g_2^D + g_4^D$  is around  $0.6 \text{ GeV}^{-1}$ , and so the coupling constant  $g_4^D$  itself is around  $1.8 \text{ GeV}^{-1}$ .

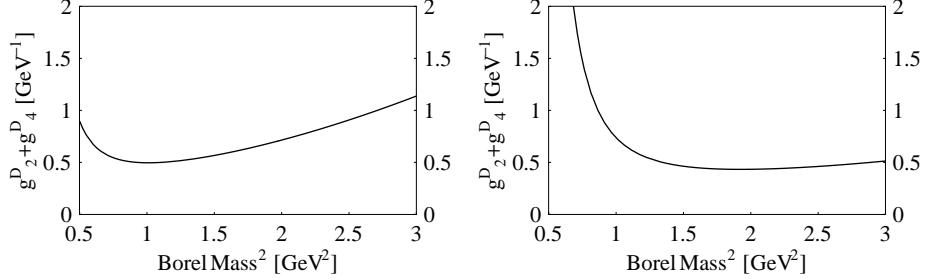


FIG. 4: The quantity  $g_2^D + g_4^D$  as a function of  $M_B^2$ . The left and right figures are obtained using Eqs. (43) and (44), respectively. The Lorentz structure is  $q_\mu q_\nu$ .

### C. Decay Width

The decay width of  $\pi_1 \rightarrow b_1 \pi$  reads:

$$\Gamma(\pi_1^0 \rightarrow b_1^+ \pi^- + b_1^- \pi^+) = 2 \times \frac{(g_{b_1 \pi}^{S'})^2}{24\pi} \frac{|\vec{q}_\pi|}{m_{\pi_1}^2} \left( 3 + \frac{|\vec{q}_\pi|^2}{m_{b_1}^2} \right) \quad (45)$$

$$+ 2 \times \frac{(g_2^D)^2}{24\pi} \frac{|\vec{q}_\pi|^5}{m_{b_1}^2} \quad (46)$$

$$+ 2 \times \frac{(g_4^D)^2}{24\pi} \frac{|\vec{q}_\pi|^5}{m_\pi^2 m_{b_1}^2} \left( 2|\vec{q}_\pi|^2 + m_{\pi_1}^2 + m_{b_1}^2 + 2\sqrt{|\vec{q}_\pi|^2 + m_{b_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} \right) \quad (47)$$

$$+ 2 \times \frac{g_{b_1 \pi}^{S'} g_2^D}{12\pi} \frac{|\vec{q}_\pi|^3}{m_{\pi_1} m_{b_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_{b_1}^2} \quad (48)$$

$$+ 2 \times \frac{g_{b_1 \pi}^{S'} g_4^D}{12\pi} \frac{|\vec{q}_\pi|^3}{m_{\pi_1} m_{b_1}^2} \left( \sqrt{|\vec{q}_\pi|^2 + m_{b_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} + |\vec{q}_\pi|^2 + m_{b_1}^2 \right) \quad (49)$$

$$+ 2 \times \frac{g_2^D g_4^D}{12\pi} \frac{|\vec{q}_\pi|^5}{m_{\pi_1} m_{b_1}^2} \left( \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} + \sqrt{|\vec{q}_\pi|^2 + m_{b_1}^2} \right), \quad (50)$$

where  $g_{b_1 \pi}^{S'}$  is defined as

$$g_{b_1 \pi}^{S'} = g_{b_1 \pi}^S + m_{\pi_1} \sqrt{m_\pi^2 + |\vec{q}_\pi|^2} g_2^D + m_\pi^2 g_4^D = -0.58 \text{ GeV}. \quad (51)$$

The decay width of  $\pi_1 \rightarrow b_1 \pi$  is around 3 MeV.

## V. THE DECAY MODE $\pi_1 \rightarrow f_1(1285)\pi$

### A. Three-Point Correlation Function

In this section we study the decay mode  $\pi_1 \rightarrow f_1(1285)\pi$ . The isoscalar axial-vector current for  $f_1(1285)$  is

$$j_\mu^{f_1}(x) = \frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d), \quad (52)$$

and it couples to  $f_1(1285)$  through [57]

$$\langle 0 | j_\mu^{f_1} | f_1(p, \lambda) \rangle = m_{f_1} f_{f_1} \epsilon_{f_1}^\mu. \quad (53)$$

Then we can write down the three-point correlation function for the decay mode  $\pi_1 \rightarrow f_1(1285)\pi$ :

$$\begin{aligned} T_{\mu\nu}^{f_1\pi(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} j_\nu^{f_1}(x) j^\pi(y) \eta_\mu^\dagger(0) | 0 \rangle \\ &= (g_{\mu\mu'} - \frac{p_\mu p_{\mu'}}{m_{\pi_1}^2}) (g_{\nu\nu'} - \frac{p'_\nu p'_{\nu'}}{m_{f_1}^2}) \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{f_1} m_{f_1} f'_\pi}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - p'^2)(m_\pi^2 - q^2)} \\ &\quad \times \left( g_{f_1\pi}^S g_{\mu'\nu'} + g_1^D ((p \cdot p') g_{\mu'\nu'} - p'_{\mu'} p_{\nu'}) + g_2^D ((p \cdot q) g_{\mu'\nu'} - q_{\mu'} p_{\nu'}) \right. \\ &\quad \left. + g_3^D ((p' \cdot q) g_{\mu'\nu'} - p'_{\mu'} q_{\nu'}) + g_4^D (q^2 g_{\mu'\nu'} - q_{\mu'} q_{\nu'}) \right). \end{aligned} \quad (54)$$

We note here that the current  $j_\mu^{f_1}$  can also couple to the pseudoscalar meson  $\eta$  and  $\eta'$  through

$$\langle 0 | j_\mu^{f_1} | \eta(p), \eta'(p) \rangle = i f_{\eta, \eta'} p_\mu. \quad (55)$$

Now the three-point correlation function is:

$$T_{\mu\nu}^{f_1\pi(\text{PH})}(p, p', q) = g'_{\eta\pi} p'_\nu (g_{\eta\pi}^C q_\alpha + g_{\eta\pi}^D p'_\alpha) (g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_{\pi_1}^2}) \frac{\sqrt{2} i f_{\pi_1} m_{\pi_1}^3 f_{\eta, \eta'} f'_\pi}{(m_{\pi_1}^2 - p^2)(m_{\eta, \eta'}^2 - p'^2)(m_\pi^2 - q^2)}. \quad (56)$$

Only Eq. (54) contains the Lorentz structures  $g_{\mu\nu}$ ,  $q_\mu q_\nu$  and  $p'_\mu q_\nu$ , which can be used to differentiate them.

At the quark and gluon level, we obtain the following OPE which is divergent at  $q^2 \rightarrow 0$ :

$$\begin{aligned} T_{\mu\nu}^{f_1\pi(\text{OPE})}(p, p', q) &= g_{\mu\nu} \times \left( \frac{i \langle g_s \bar{q} \sigma G q \rangle}{6\sqrt{2}} \left( \frac{3p \cdot q}{p^2 q^2} + \frac{p' \cdot q}{p'^2 q^2} \right) + \frac{i \langle \bar{q} q \rangle \langle g_s^2 G G \rangle}{18\sqrt{2}} \left( -\frac{p \cdot q}{p^4 q^2} + \frac{p \cdot q}{p^2 q^4} - \frac{p' \cdot q}{p'^4 q^2} \right. \right. \\ &\quad \left. \left. + \frac{p' \cdot q}{p'^2 q^4} \right) + \frac{i \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 G G \rangle}{48\sqrt{2}} \left( -\frac{1}{p'^4 q^2} + \frac{1}{p'^2 q^4} + \left( \frac{1}{p'^6 q^4} - \frac{1}{p'^4 q^6} \right) \frac{8}{3} (p' \cdot q)^2 \right) \right) \\ &\quad + q_\mu q_\nu \times \left( -\frac{i \langle g_s \bar{q} \sigma G q \rangle}{\sqrt{2}} \frac{1}{p^2 q^2} + \frac{i \langle \bar{q} q \rangle \langle g_s^2 G G \rangle}{18\sqrt{2}} \frac{1}{p^4 q^2} \right) \\ &\quad + p'_\mu q_\nu \times \left( -\frac{i \langle g_s \bar{q} \sigma G q \rangle}{6\sqrt{2}} \left( \frac{1}{p'^2 q^2} + \frac{3}{p^2 q^2} \right) \right). \end{aligned} \quad (57)$$

We note that the  $\alpha_s$  correction vanishes in this case.

## B. Numerical Analysis

We use the following values to perform the numerical analysis [57]:

$$m_{f_1} = 1285 \text{ MeV}, f_{f_1} = 170 \text{ MeV}. \quad (58)$$

where  $f_{f_1}$  is obtained using the sum rules Eq. (4.52) in Ref. [57] when assuming  $m_{f_1} = 1285$  MeV. Comparing Eqs. (34) and (37), we find several Lorentz structures, and we can obtain several different QCD sum rules. They are  $g_{\mu\nu}$ ,  $p'_\mu p_\nu$ ,  $q_\mu p_\nu$ ,  $p'_\mu q_\nu$ , and  $q_\mu q_\nu$ .

### 1. Lorentz Structure $g_{\mu\nu}$

First we choose the Lorentz structure  $g_{\mu\nu}$ . After performing the Borel transformation once, we obtain

$$\begin{aligned} -g_{f_1\pi}^S \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{f_1} m_{f_1} f'_\pi}{m_{f_1}^2 - m_{\pi_1}^2} (e^{-m_{\pi_1}^2/M^2} - e^{-m_{f_1}^2/M^2}) - g_1^D \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{f_1} m_{f_1} f'_\pi}{m_{f_1}^2 - m_{\pi_1}^2} (m_{\pi_1} e^{-m_{\pi_1}^2/M^2} - m_{f_1} e^{-m_{f_1}^2/M^2}) \\ = -\frac{5i \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 G G \rangle}{144\sqrt{2}} \frac{1}{T^2}. \end{aligned} \quad (59)$$

Here the dominant condensate  $\langle g_s \bar{q} \sigma G q \rangle$  vanishes after the Borel transformation, and so the coupling constant  $g_{f_1\pi}^S$  is calculated to be around 0.05 GeV, which is much smaller than  $g_3^D$  and  $g_4^D$  which we will study in the following

subsections. So we are in the same situation as the previous section. Similarly we omit some small terms, and the three-point correlation function (54) is simplified to be

$$T_{\mu\nu}^{f_1\pi(\text{PH})}(p, p', q) = \frac{\sqrt{2}f_{\pi_1}m_{\pi_1}^3f_{f_1}m_{f_1}f'_\pi}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - p'^2)(m_\pi^2 - q^2)} \times \left( g_{\mu\nu}(g_{b_1\pi}^S + g_1^D(p \cdot p') + g_2^D(p \cdot q) + g_3^D(p' \cdot q) + g_4^Dq^2) - g_1^Dp'_\mu p'_\nu - g_2^Dq_\mu p'_\nu - (g_1^D + g_3^D)p'_\mu q_\nu - (g_2^D + g_4^D)q_\mu q_\nu + \dots \right). \quad (60)$$

### 2. Lorentz Structures $p'_\mu p'_\nu$ and $q_\mu p'_\nu$

For the Lorentz structures  $p'_\mu p'_\nu$  and  $q_\mu p'_\nu$ , the OPE side vanishes, and so we simply obtain

$$g_1^D = 0, \text{ and } g_2^D = 0. \quad (61)$$

### 3. Lorentz Structures $p'_\mu q_\nu$

In this subsection we choose the Lorentz structure  $p'_\mu q_\nu$ . After performing the Borel transformation once, we obtain

$$g_3^D \frac{\sqrt{2}f_{\pi_1}m_{\pi_1}^3f_{f_1}m_{f_1}f'_\pi}{m_{f_1}^2 - m_{\pi_1}^2} (e^{-m_{\pi_1}^2/M^2} - e^{-m_{f_1}^2/M^2}) = \frac{2i\langle g_s \bar{q} \sigma G q \rangle}{3\sqrt{2}}. \quad (62)$$

The coupling constant  $g_3^D$  is a function of  $M_B$ , as shown in Fig. 5. The result is around 5 GeV $^{-1}$ .

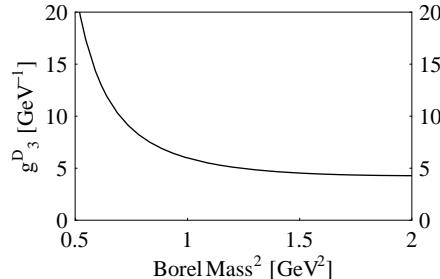


FIG. 5: The coupling constant  $g_3^D$  as a function of  $M_B^2$ , obtained using Eq. (62). The Lorentz structure is  $p'_\mu q_\nu$ .

### 4. Lorentz Structures $q_\mu q_\nu$

In this subsection we choose the Lorentz structure  $q_\mu q_\nu$ . After performing the Borel transformation once, we obtain

$$g_4^D \frac{\sqrt{2}f_{\pi_1}m_{\pi_1}^3f_{f_1}m_{f_1}f'_\pi}{m_{f_1}^2 - m_{\pi_1}^2} (e^{-m_{\pi_1}^2/M^2} - e^{-m_{f_1}^2/M^2}) = \frac{i\langle g_s \bar{q} \sigma G q \rangle}{\sqrt{2}} + \frac{i\langle \bar{q} q \rangle \langle g_s^2 G G \rangle}{18\sqrt{2}} \frac{1}{T^2}. \quad (63)$$

The coupling constant  $g_4^D$  is a function of  $M_B$ , as shown in Fig. 6. The result is around 7 GeV $^{-1}$ .

## C. Decay Width

To calculate the decay width of  $\pi_1 \rightarrow f_1 \pi$ , we similarly introduce

$$g_{f_1\pi}^{S'} = g_{f_1\pi}^S + (\sqrt{m_{f_1}^2 + |\vec{q}_\pi|^2} \sqrt{m_\pi^2 + |\vec{q}_\pi|^2} + |\vec{q}_\pi|^2) g_2^D + m_\pi^2 g_4^D = 2.41 \text{ GeV}. \quad (64)$$

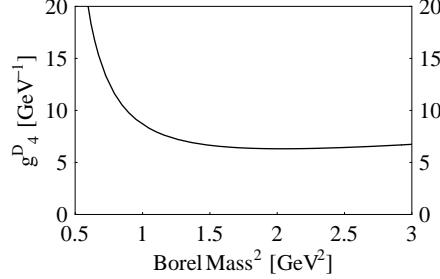


FIG. 6: The coupling constant  $g_4^D$  as a function of  $M_B^2$ , obtained using Eq. (63). The Lorentz structure is  $q_\mu q_\nu$ .

The decay width reads

$$\begin{aligned} & \Gamma(\pi_1^0 \rightarrow f_1 \pi^0) \\ &= \frac{(g_{f_1 \pi}^{S'})^2}{24\pi} \frac{|\vec{q}_\pi|}{m_{\pi_1}^2} \left( 3 + \frac{|\vec{q}_\pi|^2}{m_{f_1}^2} \right) \end{aligned} \quad (65)$$

$$+ \frac{(g_3^D)^2}{24\pi} \frac{|\vec{q}_\pi|^5}{m_\pi^2 m_{f_1}^2} \left( 2|\vec{q}_\pi|^2 + m_{\pi_1}^2 + m_{f_1}^2 + 2\sqrt{|\vec{q}_\pi|^2 + m_{f_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} \right) \quad (66)$$

$$+ \frac{(g_4^D)^2}{24\pi} \frac{|\vec{q}_\pi|^5}{m_\pi^2 m_{f_1}^2} \left( 2|\vec{q}_\pi|^2 + m_{\pi_1}^2 + m_{f_1}^2 + 2\sqrt{|\vec{q}_\pi|^2 + m_{f_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} \right) \quad (67)$$

$$- \frac{g_{f_1 \pi}^{S'} g_3^D}{12\pi} \frac{|\vec{q}_\pi|^3}{m_{\pi_1}^2 m_{f_1}^2} \left( \sqrt{|\vec{q}_\pi|^2 + m_{f_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} + |\vec{q}_\pi|^2 + m_{f_1}^2 \right) \quad (68)$$

$$+ \frac{g_{f_1 \pi}^{S'} g_4^D}{12\pi} \frac{|\vec{q}_\pi|^3}{m_{\pi_1}^2 m_{f_1}^2} \left( \sqrt{|\vec{q}_\pi|^2 + m_{f_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} + |\vec{q}_\pi|^2 + m_{f_1}^2 \right) \quad (69)$$

$$- \frac{g_3^D g_4^D}{12\pi} \frac{|\vec{q}_\pi|^5}{m_\pi^2 m_{f_1}^2} \left( 2|\vec{q}_\pi|^2 + m_{\pi_1}^2 + m_{f_1}^2 + 2\sqrt{|\vec{q}_\pi|^2 + m_{f_1}^2} \sqrt{|\vec{q}_\pi|^2 + m_\pi^2} \right). \quad (70)$$

Here (65) mainly comes from the  $S$ -wave decay, (66) and (67) come from the  $D$ -wave decays, and the last three are the  $S$  wave and  $D$  wave interference terms. Numerically they are:

$$(65) = 23.2 \text{ MeV}, (66) = 0.4 \text{ MeV}, (67) = 0.8 \text{ MeV}, (68) = -2.6 \text{ MeV}, (69) = 3.6 \text{ MeV}, (70) = -1.2 \text{ MeV},$$

The decay width of  $\pi_1 \rightarrow f_1 \pi$  is around 24 MeV.

Finally we want to note two important facts here. First, all the three Lorentz structures  $g_{\mu\nu}$ ,  $q_\mu q_\nu$  and  $p'_\mu q_\nu$  which lead to non-zero coupling constants are not contaminated by the other decay mode  $\eta\pi$ , as shown in Eq. (56). Secondly, from Eq. (57) we find that all these three different Lorentz structures contain the condensate  $\langle g_s \bar{q} \sigma G q \rangle$ , the dominant one. However, for the Lorentz structure  $g_{\mu\nu}$ , it vanishes after the Borel transformation. So the obtained  $g_{f_1 \pi}^S$  is much smaller than  $g_3^D$  and  $g_4^D$  obtained using the other two Lorentz structures, where this condensate does not vanish.

## VI. ISOSCALAR HYBRID

In this section, we turn to study the decay modes of the isoscalar hybrid state  $\sigma_1$  with the quantum numbers  $I^G J^{PC} = 0^+ 1^{-+}$ . Now the  $S$ -wave decay modes are  $a_1(1260)\pi$  and  $f_1(1285)\eta$ , etc., while the  $P$ -wave decay modes are  $\eta\eta'$ , etc. The interpolating current for  $\sigma_1$  is

$$\psi_\mu = \frac{1}{\sqrt{2}} (\bar{u}^a \gamma_\nu u^b + \bar{d}^a \gamma_\nu d^b) \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n, \quad (71)$$

and it couples to the isoscalar hybrid state  $\sigma_1$  through

$$\langle 0 | \psi_\mu | \sigma_1 \rangle = \sqrt{2} f_{\sigma_1} m_{\sigma_1}^3 \epsilon_\mu^\lambda. \quad (72)$$

The QCD sum rule leads to the following relations

$$\begin{aligned} T_{\text{OPE}}^{\sigma_1 \rightarrow a_1 \pi}(p, p', q) &= T_{\text{OPE}}^{\pi_1 \rightarrow f_1 \pi}(p, p', q), \\ T_{\text{OPE}}^{\sigma_1 \rightarrow f_1 \eta}(p, p', q) &= i\left(\frac{1}{\sqrt{3}} \cos \theta_P - \sqrt{\frac{2}{3}} \sin \theta_P\right) \times T_{\text{OPE}}^{\pi_1 \rightarrow f_1 \pi}(p, p', q), \\ T_{\text{OPE}}^{\sigma_1 \rightarrow \eta \eta'}(p, p', q) &= i\left(\frac{1}{\sqrt{3}} \sin \theta_P + \sqrt{\frac{2}{3}} \cos \theta_P\right) \times T_{\text{OPE}}^{\pi_1 \rightarrow \eta \pi}(p, p', q). \end{aligned} \quad (73)$$

From these relations, the decay width of  $\sigma_1 \rightarrow a_1(1260)\pi$ ,  $f_1(1285)\eta$ , and  $\eta\eta'$  can be easily obtained. For example, with  $m_{\sigma_1} = 2.0$  GeV and  $f_{\sigma_1} = 0.013$  GeV,  $m_{a_1} = 1.26$  GeV,  $f_{a_1} = 0.17$  GeV, the decay width of  $\sigma_1 \rightarrow a_1(1260)\pi$ ,  $f_1(1285)\eta$ , and  $\eta\eta'$  are around 770 MeV, 74 MeV and 0.3 MeV, respectively.

## VII. SUMMARY

We have studied the decay properties of the hybrid states with  $J^{PC} = 1^{-+}$  using the method of QCD sum rule. We have used the three-point correlation functions to extract the coupling constants, and then calculated the relevant decay widths. We work at the pion pole. First we take the  $\pi$  mass in the denominator to be zero. Then we work at the limit  $q^2 \rightarrow 0$  and pick out the divergent terms only. Such a procedure simplifies the calculation greatly at the cost of throwing away all the finite pieces.

TABLE I: The decay widths of the isovector hybrid state  $\pi_1$  of  $I^G J^{PC} = 1^- 1^{-+}$  and isoscalar one  $\sigma_1$  of  $I^G J^{PC} = 0^+ 1^{-+}$  when the hybrid mass is taken to be 1.6 GeV, 1.8 GeV and 2.0 GeV respectively.

Decay Modes	Decay Angular Momentum	Widths (MeV)		
		$M = 1.6$ GeV	$M = 1.8$ GeV	$M = 2.0$ GeV
$\pi_1 \rightarrow \rho\pi$	$P$ -wave	180 MeV	410 MeV	800 MeV
$\pi_1 \rightarrow \eta\pi$	$P$ -wave	0.9 MeV	2 MeV	4 MeV
$\pi_1 \rightarrow \eta'\pi$	$P$ -wave	0.1 MeV	0.4 MeV	0.9 MeV
$\pi_1 \rightarrow b_1\pi$	$S+D$ -waves	2.9 MeV	14 MeV	53 MeV
$\pi_1 \rightarrow f_1\pi$	$S+D$ -waves	24 MeV	140 MeV	410 MeV
$\sigma_1 \rightarrow \eta\eta'$	$P$ -wave	< 0.1 MeV	0.1 MeV	0.3 MeV
$\sigma_1 \rightarrow a_1\pi$	$S+D$ -waves	60 MeV	310 MeV	770 MeV
$\sigma_1 \rightarrow f_1\pi$	$S+D$ -waves	—	—	74 MeV

We have calculated the decay widths of both the isovector hybrid state  $\pi_1$  with  $I^G J^{PC} = 1^- 1^{-+}$  and isoscalar one  $\sigma_1$  with  $I^G J^{PC} = 0^+ 1^{-+}$ . The present three-point correlation function formalism works well for the decay processes  $\pi_1 \rightarrow \rho\pi$ ,  $\pi_1 \rightarrow \eta\pi$  and  $\pi_1 \rightarrow \eta'\pi$  where the Lorentz structures are relatively simple. We have also discussed the modes  $b_1\pi$ ,  $f_1\pi$ . These two decay modes are complicated by the many Lorentz structures in the three-point correlation function, possible mixing between the S-wave and D-wave decay patterns, and possible contamination from other decay modes such as  $\rho\pi$ . We illustrate the variation of the decay width with the hybrid meson mass in Table I. The S-wave decay width of the  $b_1\pi$ ,  $f_1\pi$  modes increases very quickly as the hybrid meson mass and decay momentum increase. But for the low mass hybrid meson around 1.6 GeV, the  $\rho\pi$  mode is one of the dominant decay modes.

For the  $1^{-+}$  state  $\pi_1(1600)$ , the decay widths of  $\pi_1 \rightarrow \rho\pi$ ,  $\pi_1 \rightarrow \eta\pi$  and  $\pi_1 \rightarrow \eta'\pi$  are around 180 MeV, 0.9 MeV and 0.1 MeV, respectively. The modes  $\eta\pi$ ,  $\eta'\pi$  are strongly suppressed compared with the  $\rho\pi$  mode. Moreover, the  $\rho\pi$  mode is one of the dominant decay modes of the  $\pi_1(1600)$ , which is in strong contrast with predictions from some phenomenological models.

In this paper we use the hybrid currents to study these hybrid states. We can also use the tetraquark currents which may also couple to these  $1^{-+}$  states. We have used such currents to study their masses, and found their possible decay modes through Fierz transformation [39, 40]. We plan to study their decay properties like their decay widths etc in the future. This might be useful to know the internal structure of the  $1^{-+}$  states.

We suggest the experimental search of  $\pi_1(1600)$  through the decay chains at BESIII:  $e^+e^- \rightarrow J/\psi(\psi') \rightarrow \pi_1 + \gamma$  or  $e^+e^- \rightarrow J/\psi(\psi') \rightarrow \pi_1 + \rho$  where the  $\pi_1$  state can be reconstructed through the decay modes  $\pi_1 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$  or  $\pi_1 \rightarrow f_1(1285)\pi^0$ . More details of possible experimental search at BESIII can be found in Ref. [70]. It is also interesting to look for  $\pi_1$  using the available BELLE/BABAR data through the process  $e^+e^- \rightarrow \gamma^* \rightarrow \rho\pi_1, b_1\pi_1, \gamma\pi_1$  etc. Hopefully our result will be helpful to the experimental identification of the  $1^{-+}$  state.

### Acknowledgments

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## Appendix A: The Decay Mode $\pi_1 \rightarrow b_1(1235)\pi$ With the Tensor Current

### 1. Three-Point Correlation Function

In this appendix, we consider the tensor current for the  $b_1(1235)$  meson and study the decay mode  $\pi_1 \rightarrow b_1(1235)\pi$ :

$$j_{\mu\nu}^{\text{TE}+} = \bar{d}\sigma_{\mu\nu}u, j_{\mu\nu}^{\text{TE}0} = \frac{1}{\sqrt{2}}(\bar{u}\sigma_{\mu\nu}u - \bar{d}\sigma_{\mu\nu}d), j_{\mu\nu}^{\text{TE}-} = \bar{u}\sigma_{\mu\nu}d, \quad (\text{A1})$$

which couples to  $b_1(1235)$  through [68]

$$\langle 0 | j_{\mu\nu}^{\text{TE}} | b_1(p, \lambda) \rangle = i f_{b_1}^{\text{TE}} \epsilon_{\mu\nu\alpha\beta} \epsilon_{\lambda}^{\alpha} p^{\beta}. \quad (\text{A2})$$

The three-point correlation function at the phenomenological side is:

$$\begin{aligned} T_{\mu\rho\sigma}^{\text{TE}(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} j_{\rho\sigma}^{\text{TE}-}(x) j^{\pi+}(y) \eta_{\mu}^{\dagger}(0) | 0 \rangle \\ &= \epsilon_{\rho\sigma\alpha\beta} p'^{\beta} \left( g_{\mu\mu'} - \frac{p_{\mu}p_{\mu'}}{m_{\pi_1}^2} \right) \left( g_{\alpha\alpha'} - \frac{p'_{\alpha}p'_{\alpha'}}{m_{b_1}^2} \right) \frac{\sqrt{2}i f_{\pi_1} m_{\pi_1}^3 f_{b_1}^{\text{TE}} f'_{\pi}}{(m_{\pi_1}^2 - p^2)(m_{b_1}^2 - p'^2)(m_{\pi}^2 - q^2)} \\ &\quad \times \left( g_{b_1\pi}^{\text{S}} g_{\mu'\nu'} + g_1^{\text{D}} ((p \cdot p') g_{\mu'\nu'} - p'_{\mu'} p_{\nu'}) + g_2^{\text{D}} ((p \cdot q) g_{\mu'\nu'} - q_{\mu'} p_{\nu'}) \right. \\ &\quad \left. + g_3^{\text{D}} ((p' \cdot q) g_{\mu'\nu'} - p'_{\mu'} q_{\nu'}) + g_4^{\text{D}} (q^2 g_{\mu'\nu'} - q_{\mu'} q_{\nu'})) \right). \end{aligned} \quad (\text{A3})$$

Unfortunately, the tensor current can also couple to the vector meson  $\rho(770)$  through [68]

$$\langle 0 | j_{\mu\nu}^{\text{TE}} | \rho(p, \lambda) \rangle = i f_{\rho}^{\text{TE}} (\epsilon_{\mu}^{\lambda} p_{\nu} - \epsilon_{\nu}^{\lambda} p_{\mu}). \quad (\text{A4})$$

When the tensor current couples to  $\rho$ , the three-point correlation function is

$$\begin{aligned} T_{\mu\rho\sigma}^{\text{TE}(\text{PH})}(p, p', q) &= \int d^4x d^4y e^{ip'x} e^{iqy} \langle 0 | \mathbb{T} j_{\rho\sigma}^{\text{TE}-}(x) j^{\pi+}(y) \eta_{\mu}^{\dagger}(0) | 0 \rangle \\ &= (g_{\mu\mu'} - \frac{p_{\mu}p_{\mu'}}{m_{\pi_1}^2}) \frac{\sqrt{2}i f_{\pi_1} m_{\pi_1}^3 f_{\rho}^{\text{TE}} f'_{\pi}}{(m_{\pi_1}^2 - p^2)(m_{\rho}^2 - p'^2)(m_{\pi}^2 - q^2)} \\ &\quad \times g_{\rho\pi}^{\text{TE}} \epsilon_{\mu'\nu'\alpha\beta} q^{\alpha} p'^{\beta} \left( (g_{\rho\nu'} - \frac{p'_{\rho}p'_{\nu'}}{m_{\rho}^2}) p'_{\sigma} - (g_{\sigma\nu'} - \frac{p'_{\sigma}p'_{\nu'}}{m_{\rho}^2}) p'_{\rho} \right). \end{aligned} \quad (\text{A5})$$

We note that the following relation exists between these Lorentz structures

$$\epsilon_{\mu\nu\alpha\beta} q^{\alpha} p^{\beta} p_{\rho} + \epsilon_{\nu\rho\alpha\beta} q^{\alpha} p^{\beta} p_{\mu} + \epsilon_{\rho\mu\alpha\beta} q^{\alpha} p^{\beta} p_{\nu} = -\epsilon_{\mu\nu\rho\alpha} q^{\alpha} p^2 + \epsilon_{\mu\nu\rho\alpha} p^{\alpha} p \cdot q. \quad (\text{A6})$$

And we can write Eq. (A5) as

$$T_{\mu\rho\sigma}^{\text{TE}(\text{PH})}(p, p', q) = g_{\rho\pi}^{\text{TE}} \epsilon_{\mu\rho\sigma\alpha} p^{\alpha} p \cdot q \frac{\sqrt{2}i f_{\pi_1} m_{\pi_1}^3 f_{\rho}^{\text{T}} f'_{\pi}}{(m_{\pi_1}^2 - p^2)(m_{\rho}^2 - p'^2)(m_{\pi}^2 - q^2)} + \dots \quad (\text{A7})$$

The contribution of  $\rho\pi$  appears in our analysis at the leading Lorentz structure  $\epsilon_{\mu\rho\sigma\alpha} p'^{\alpha}$ , and makes it difficult to single out the contribution of  $b_1\pi$ .

At the quark and gluon level, we obtain the following OPE which is divergent at  $q^2 \rightarrow 0$

$$\begin{aligned} q^2 T_{\mu\rho\sigma}^{\text{TE(OPE)}}(p, p', q) \rightarrow & \epsilon_{\rho\sigma\alpha\beta} p'^\alpha q^\beta q_\mu \frac{i \langle g_s^2 GG \rangle}{24\sqrt{2}\pi^2} \int_0^\infty e^{(\tau_1+\tau_2)p^2} \frac{\tau_2}{(\tau_1+\tau_2)^2} d\tau_1 d\tau_2 \\ & + \frac{i \langle g_s^3 fGGG \rangle}{192\sqrt{2}\pi^2} \int_0^\infty e^{(\tau_1+\tau_2)p^2} \left( \epsilon_{\mu\rho\sigma\alpha} q^\alpha \left( \frac{8\tau_1 - 6\tau_2}{(\tau_1+\tau_2)^2} + p'^2 \frac{-8\tau_2^2}{(\tau_1+\tau_2)^2} \right) \right. \\ & + \epsilon_{\mu\rho\sigma\alpha} p'^\alpha \frac{-20\tau_2}{(\tau_1+\tau_2)^2} + \epsilon_{\rho\sigma\alpha\beta} p'^\alpha q^\beta q_\mu \frac{-4\tau_2(\tau_1^2 + \tau_2^2)}{(\tau_1+\tau_2)^3} \\ & \left. + \epsilon_{\mu\rho\sigma\alpha} p'^\alpha (p^2 - p'^2) \frac{2\tau_2^2}{(\tau_1+\tau_2)^2} \right) d\tau_1 d\tau_2 - \frac{3i \langle g_s^3 fGGG \rangle}{32\sqrt{2}\pi^2} \frac{\epsilon_{\mu\rho\sigma\alpha} p^\alpha}{p^2}. \end{aligned} \quad (\text{A8})$$

## 2. Numerical Analysis

In the numerical analysis, we use the following value [68]:

$$f_{b_1}^{\text{TE}}(2\text{GeV}) = 180(20)\text{MeV}. \quad (\text{A9})$$

The leading Lorentz structure  $\epsilon_{\mu\rho\sigma\alpha} p'^\alpha$  appears in the  $S$ -wave decay. Both  $\rho\pi$  and  $b_1\pi$  modes contribute to this structure. If we naively assume that only the  $b_1\pi$  mode contributes, we obtain after performing the Borel transformation once

$$-g_{b_1\pi}^S \frac{\sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{b_1}^{\text{TE}} f_\pi'}{m_{b_1}^2 - m_{\pi_1}^2} \left( e^{-m_{\pi_1}^2/T^2} - e^{-m_{b_1}^2/T^2} \right) = \frac{7i \langle g_s^3 fGGG \rangle}{48\sqrt{2}\pi^2}. \quad (\text{A10})$$

We can also perform the Borel transformation twice, and then obtain

$$-g_{b_1\pi}^S \sqrt{2} f_{\pi_1} m_{\pi_1}^3 f_{b_1}^{\text{TE}} f_\pi' e^{-m_{\pi_1}^2/T_1^2} e^{-m_{b_1}^2/T_2^2} = \frac{i \langle g_s^3 fGGG \rangle}{192\sqrt{2}\pi^2} \frac{-20\tau_2}{(\tau_1+\tau_2)^2} \Big|_{\tau_1=1/T_1^2, \tau_2=1/T_2^2}. \quad (\text{A11})$$

Using Eqs. (A10) and (A11) we can perform the numerical analysis. The extracted  $g_{b_1\pi}^S$  is around 0.5 GeV as shown in Fig. 7, which is significantly larger than our previous result. We do not use this value to calculate the decay width since we are unable to subtract the contamination from the decay mode  $\rho\pi$  very cleanly. One may use the extracted  $\rho\pi$  coupling constant in the previous sections as input and subtract such contribution from the sum rule derived using the tensor current. However, such an approach is very crude with large uncertainty.

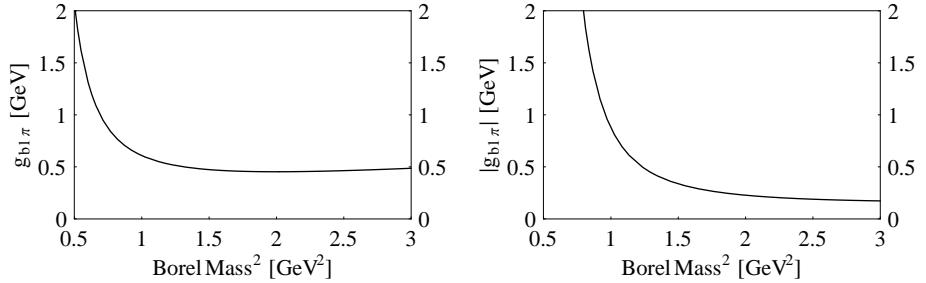


FIG. 7: The coupling constant  $g_{b_1\pi}^S$  as a function of  $M_B^2$ . The left and right figures are obtained using Eqs. (A10) and (A11), respectively. The Lorentz structure is  $\epsilon_{\mu\rho\sigma\alpha} p'^\alpha$ .

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